Refutation of a Gerhard W. Bruhn paper

M.W. Evans

Alpha Institute for Advanced Studies (AIAS) (www.aias.us)

The following is a refutation of the paper: Bruhn, G. W. (2006), *The Central Error of Myron W. Evans' ECE Theory - a Type Mismatch* [export.arxiv.org/pdf/physics/0607190v1].

Detailed Points of Refutation

Bruhn starts by stating that "M.W. Evans constructs his spacetime by a dubious alternative method … Here we sketch the usual method of constructing the 4-dimensional spacetime manifold *M*".

However, that "dubious alternative method" is the same method that is taught in universities. In fact, the definitions (1.2), (1.7) and (1.8) that Bruhn identifies as "the usual method" are the *same* definitions that are used in ECE theory. So, what is Bruhn trying to say?

In Section 2, the assertion by Bruhn in going from his Eq. (2.1) to (2.2) is *not* made in ECE theory. The correct method, which was already given in Paper 12, Section 2, is repeated below.

Start with the Einstein Field Equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\mu\nu} \quad . \tag{1}$$

Then introduce:

$$R_{\mu\nu} = R^a_\mu q^b_\nu \eta_{ab} \quad , \tag{2}$$

$$T_{\mu\nu} = T^a_\mu q^b_\nu \eta_{ab} \quad , \tag{3}$$

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab} \quad . \tag{4}$$

Eq. (4) is the standard decomposition of the metric into the product of two tetrads (e.g., S. M. Carroll, *Lecture Notes on General Relativity*, arxiv.org/pdf/gr-qc/9712019.pdf).

This standard decomposition is used in Eqs. (2) and (3) to define R^a_μ and T^a_μ , which are vector valued one forms. One of the Evans field equations is

$$G^a_\mu = -\frac{1}{4} R q^a_\mu \quad , \tag{5}$$

$$T^a_\mu = \frac{1}{4} T q^a_\mu \quad . \tag{6}$$

Eqs. (5) and (6) are derived from the definitions of R and T that were originally used by Einstein:

$$R = g^{\mu\nu} R_{\mu\nu} \ \ , \ \ T = g^{\mu\nu} T_{\mu\nu} \ \ . \tag{7}$$

Using the Einstein convention:

$$g^{\mu\nu} g_{\mu\nu} = 4 \ , \tag{8}$$

and the Cartan convention:

$$q^a_\mu q^\mu_a = 1 \ , \tag{9}$$

we obtain

$$R = g^{\mu\nu} R_{\mu\nu} = q^a_\mu q^\nu_b \eta^{ab} R^a_\mu q^b_\nu \eta_{ab} \ \ , \tag{10}$$

where we have used Eq. (2).

We multiply both sides of Eq. (10) by q^a_μ to obtain

$$R^a_\mu = \frac{1}{4} R q^a_\mu \ \ . \tag{11}$$

This is because the right side of Eq. (10) is

$$\left(\eta^{ab}\eta_{ab}\right)\left(q^\nu_b q^b_\nu\right)\left(q^a_\mu R^a_\mu\right) = 4 q^a_\mu R^a_\mu \ \ . \tag{12}$$

Then multiply both sides of Eq. (12) by q^a_μ to obtain

$$R^a_\mu = \frac{1}{4} R q^a_\mu \ \ . \tag{13}$$

Consequently,

$$G^a_\mu = R^a_\mu - \frac{1}{2} R q^a_\mu = -\frac{1}{4} R q^a_\mu \ \ . \tag{14}$$

This is Eq. (5). Q.E.D.

Continuing with Bruhn's Section 2:

The contracted form of Eq. (1) is

$$R = -kT \tag{15}$$

(Einstein, *The Meaning of Relativity*).

Multiply both sides of Eq. (15) by q^a_μ , to obtain

$$R q^a_\mu = -kT q^a_\mu \ \ . \tag{16}$$

Substituting Eqs. (5) and (6) into Eq. (16) gives

$$G_\mu^a = kT_\mu^a \quad . \tag{17}$$

Write Eq. (17) in the following form:

$$\frac{1}{4}Rq_\mu^a - \frac{1}{2}Rq_\mu^a = \frac{1}{4}kTq_\mu^a \quad . \tag{18}$$

Multiply both sides of Eq (18) by $q_\nu^b \eta_{ab}$ to obtain

$$\frac{1}{4}Rq_\mu^a q_\nu^b \eta_{ab} - \frac{1}{2}Rq_\mu^a q_\nu^b \eta_{ab} = \frac{1}{4}kTq_\mu^a q_\nu^b \eta_{ab} \quad . \tag{19}$$

By using Eqs. (2) - (4) and (5) and (6), with Eq. (19), we get Eq. (15). Q.E.D.

This has been worked out many times before.

Starting from Eq. (16), we may construct:

$$Rq_\mu^a q_\nu^b \eta_{ab} = -kTq_\mu^a q_\nu^b \eta_{ab} \quad \text{, i.e.,} \tag{20}$$

$$Rg_{\mu\nu} = -kTg_{\mu\nu} \quad . \tag{21}$$

We may also construct the wedge product:

$$Rq_\mu^a \wedge q_\nu^b = -kTq_\mu^a \wedge q_\nu^b \quad . \tag{22}$$

The remark on Page 3 of Bruhn's paper has also been corrected many times before.

We may define the quantity

$$R_{\mu\nu}^{c(A)} := Rq_\mu^a \wedge q_\nu^b \quad , \tag{23}$$

analogously to

$$q_{\mu\nu}^{c(A)} := q_\mu^a \wedge q_\nu^b \quad . \tag{24}$$

I assume that this is what Bruhn is trying to say.

The definition of wedge product that I use is the same as that used by everyone else (e.g., S. M. Carroll, ibid.), and is given in detail in Paper 15, Appendix C, Eq. (C.5):

$$(A \wedge B)_{\mu_1 \cdots \mu_{p+q}} = \frac{(p+q)!}{p!q!}(p+1)A_{[\mu_1 \cdots \mu_p} B_{\mu_{p+1} \cdots \mu_{p+q}]} \quad . \tag{25}$$

Examples are given as Eqs. (C. 11) and (C.13).

This shows that there is no type mismatch in Eqs. (23) and (24), and no “illegal removal” of indices.

For example, there is no type mismatch in Evans’ Eq. (26), since the dimensions of $G^{(0)}$ are not specified and have to be chosen in a way that allows the dimensions of the tensor G to come out properly, which is a common procedure in physics. Furthermore, Bruhn does not seem to be aware that the form indices are often assumed implicitly in mathematical papers.

Additional mistakes by Bruhn:

a) The line element is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad , \tag{26}$$

which is a scalar, and not a symmetric two-form. Bruhn has confused a scalar with a symmetric two-form.

b) Bruhn incorrectly asserts that the Hodge dual of a two-form cannot be defined. This is, I assume, what he is trying to do on his Page 5. It is well known that the Hodge dual of a two-form is 4-D, which is another two-form (e.g., S. M. Carroll, ibid.).

c) There is also misinformation by Bruhn concerning tensors $q^{\mu\nu(S)}$ and $q^{\mu\nu(A)}$ on his Page 5. He misrepresents my definitions, and makes non-consequential deductions. My definitions are as follows:

$$q_{ij}^{(S)} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix}$$

$$= \begin{bmatrix} h_1^2 & h_1 h_2 & h_1 h_3 \\ h_2 h_1 & h_2^2 & h_2 h_3 \\ h_3 h_1 & h_3 h_2 & h_3^2 \end{bmatrix} . \tag{27}$$

These correct definitions have nothing to do with Bruhn’s remarks.

Furthermore, the definitions of the symmetric and antisymmetric metric are based on standard differential geometry. ECE theory is based on Cartan geometry, and the remarks at the foot of his Page 5 make no sense in Cartan geometry (and the ones on Page 6 do not makes sense, either).

Also, please note that there was a misprint in Bruhn [1], where (8) and (9) are missing the stars (denoting complex conjugate), e.g., $\boldsymbol{e_1} \times \boldsymbol{e_2} = \boldsymbol{e_3^*}$.

Myron W. Evans
British Civil List Scientist